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Connecting Leptogenesis to CP Violation in Neutrino Mixings in a Tri-bimaximal Mixing model

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Abstract

We show that in a recently proposed S_3 model for tri-bimaximal mixing pattern for neutrinos, CP violating phases in neutrino mixings are directly responsible for lepton asymmetry ϵ_ℓ. In the exact tri-bimaximal limit, ϵ_ℓ is proportional to one of the Majorana phases whereas in the presence of small deviations from tri-bimaximal pattern, there are two contributions, one being proportional to the Dirac phase and the other to one of the two Majorana phases. In the second case, θ_{13} is nonzero and correlated with the deviation from maximal atmospheric mixing.

I. INTRODUCTION

Seesaw mechanism for understanding small neutrino masses[1] provides an interesting way to understand the origin of matter-anti-matter asymmetry[2] via the CP violating decay of the heavy right-handed neutrinos combined with B+L violation by electroweak sphalerons[3]. This raises the very exciting possibility that better understanding of neutrino masses and mixings may help to resolve one of the deepest mysteries of the Universe i.e. the origin of matter. A lot of attention has therefore been rightly focussed on trying to connect various ways of understanding neutrino masses with leptogenesis and obtaining constraints on seesaw scale physics, lightest neutrino masses etc.[4]. A very interesting question in this connection is whether CP violating phases in neutrino mixings that can be probed in long baseline as well as in neutrinoless double beta decay experiments are the ones that are responsible for the matter-anti-matter asymmetry. It turns out that in generic seesaw models there is no apriori connection between them and it is hoped that in a true theory of neutrino masses and mixings, such a connection may exist.

Attempts to find such models have been made in the past[5] but they usually require additional assumptions about parameters not directly related to observations to establish a direct connection between leptogenesis phase and low energy neutrino phase. We repeat that by a direct connection, we mean the phase responsible for lepton asymmetry of the Universe is the same one that appears as either a Dirac or one of the two Majorana phases in neutrino mixings. The nontriviality of this problem stems from two facts: (i) in generic seesaw models, lepton asymmetry ϵ_ℓ depends only a subset of the phases of Dirac mass matrix M_D whereas low energy phases in the neutrino mass matrix involves all of them; and (ii) the seesaw formula "scrambles" up the phases due to multiplication of matrices so that any direct connection between low and high energy phases, if they exist at all becomes difficult to discern.

In this letter, we show that in a recently proposed S_3 model[6] for tri-bimaximal neutrino mixing[7], the structure of the neutrino mass matrix is so constrained by symmetry that a direct connection between the leptogenesis phase and neutrino mixing phases emerges. Thus within the context of this model, a measurement of the neutrino CP phases would provide a direct understanding of the origin of matter. This appears to us to be an interesting result. A future direction of work would be to unify quarks into the model so that one may perhaps understand the origin of quark CP violation as well.

The motivation for our work is the recent indication that present neutrino oscillation data points to a leptonic mixing pattern given by the PMNS matrix in the so-called tri-bimaximal form[7]:

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1}$$

This form is very suggestive of an underlying symmetry of leptons. The true nature of the symmetry is however far from clear, although there are many interesting suggestions[8][9][10]. Our interest here is in an S_3 model proposed in[6] where the key flavor symmetry leading to tri-bimaximal mixing is the permutation symmetry of three leptonic families. The resulting neutrino mass matrix is characterized by only three complex parameters, whose absolute values are constrained by already existing observations. We find that (i) in the exact tri-bimaximal limit, when there is no Dirac phase, one of the two Majorana phases is directly responsible for the lepton asymmetry of the Universe; (ii) even after we include small departures from the tri-bimaximal limit, the direct connection remains – there are then two contributions to ϵ_ℓ, one being proportional to the Dirac phase and the other to one of the two Majorana phases. This direct connection is possible due to the simple form of M_D dictated by the S_3 symmetry of the model and the assumptions that in case (i) only one and in case (ii) only two right handed neutrinos dominate the seesaw formula as well as the fact there is an S_3 symmetric type II contribution to the neutrino masses in both cases. We elaborate on these points below.

This paper is organized as follows: in sec. 2, we review the salient features of the S_3 model of Ref.[6] for tri-bimaximal mixing; in section 3, we present a general discussion of leptogenesis in our model; in sec. 4, we calculate the baryon asymmetry in the exact tri-bimaximal mixing and establish the direct connection between one of the Majorana phases in the neutrino mixing and ϵ_ℓ; in sec. 5, we do the same for the case where we include deviations from tri-bimaximal limit and show the connection of ϵ_ℓ to the Dirac and the Majorana phases; we summarize our results in sec. 6.

II. THE S_3 MODEL

We start with the Majorana neutrino mass matrix whose diagonalization at the seesaw scale leads to the tri-bimaximal mixing matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} a' & b' & b' \\ b' & a'-c' & b'+c' \\ b' & b'+c' & a'-c' \end{pmatrix} \tag{2}$$

where the elements are chosen to be complex. Diagonalizing this matrix leads to the U_{PMNS} of Eq. (1) and the neutrino masses: $m_1 = a'-b'; m_2 = a'+2b'$ and $m_3 = a'-b'-2c'$. Clearly if $|a'| \simeq |b'| \ll |c'|$, we get a normal hierarchy for masses. It was pointed out in Ref.[6] that the above Majorana neutrino mass matrix can be realized in a combined type I type II seesaw model with soft-broken S_3 family symmetry for leptons. The type II contribution comes from an S_3 invariant coupling $f_{\alpha\beta}L_\alpha L_\beta\Delta$,

$$f = \begin{pmatrix} f_a & f_b & f_b \\ f_b & f_a & f_b \\ f_b & f_b & f_a \end{pmatrix} \tag{3}$$

After the triplet Higgs field Δ gets vev and decouples, its contribution to the light neutrino mass can written as

$$M_{II} = \begin{pmatrix} a' & b' & b' \\ b' & a' & b' \\ b' & b' & a' \end{pmatrix} \tag{4}$$

where $a' = \frac{v^2\sin^2\beta\lambda}{M_T} f_a$ and $b' = \frac{v^2\sin^2\beta\lambda}{M_T} f_b$. We denote M_T as the mass of the triplet Higgs and λ as the coupling constant between the triplet and doublets in the superpotential.

Coming to the type I contribution, the Dirac mass matrix for neutrinos comes from an S_3 invariant Yukawa coupling of the form:

$$\mathcal{L}_D = h_\nu[\overline{\nu_{R1}}H(L_e - L_\mu) + \overline{\nu_{R2}}H(L_\mu - L_\tau) + \overline{\nu_{R3}}H(L_\tau - L_e)] + h.c. \tag{5}$$

leading to

$$Y_\nu = \begin{pmatrix} h & -h & 0 \\ 0 & h & -h \\ -h & 0 & h \end{pmatrix}. \tag{6}$$

In the limit of $|M_{R1,R3}| \gg |M_{R2}|$, where a single right-handed neutrino dominates the type I contribution, the mixed type I+II seesaw formula

$$\mathcal{M}_\nu \;=\; M_{II} - M_D^T M_{\nu R}^{-1} M_D, \tag{7}$$

gives rise to the desired form for the neutrino Majorana mass matrix which leads to the tri-bimaximal mixing[6].

We can now do the phase counting in the model. When two of the above right-handed neutrinos decouple, there is only one Yukawa coupling. We can first redefine the phase of ν_{R2} so that its mass is real and we then redefine all the lepton doublets by a common phase which now makes the Dirac Yukawa coupling h real. One cannot then do any more phase redefinitions and we are left with two phases in the neutrino mass matrix which in this basis reside in the entries a' and b' in Eq.(4). These two phases will appear as the Majorana phases in the low energy mass matrix as we show below.

As far as the charged lepton masses are concerned, the symmetry needs to be extended to $S_3 \times (Z_2)^3$ to have a simple diagonal mass matrix and all their masses can be made real by separate independent phase redefinition of the right-handed charged leptons. No new phases enter the PMNS matrix. It turns out that the $S_3 \times (Z_2)^3$ symmetric version can also be derived from an $S_4 \times Z_2$ symmetry[11] and this also does not effect our phase counting.

Turning to the case where two of the right-handed neutrinos contribute to $\mathcal{M}_\nu$, there are three phases in the light neutrino mass matrix. This is because in this case there are two apriori complex right-handed neutrino masses and only one of them together with h can be made real by phase redefinition as in the first case. This leaves the phases of a' and b' and that of the second right handed neutrino giving a total of three phases. This case represents a deviation from the tri-bimaximal mixing with the deviation being proportional to $|M_{R2}|/|M_{R3}|$. We will show in sec. 4 that the new phase in this case appears as the Dirac phase. Let us now proceed to discuss leptogenesis in both these cases. As noted, we choose f_a, f_b, M_{R3} to be complex and h, M_{R2} to be real, and express them as $f_a = |f_a| e^{i\phi_a},\; f_b = |f_b| e^{i\phi_b}, M_{R3} = M_3 e^{-i\phi_3}$ and $M_{R2} = M_2$.

III. LEPTOGENESIS IN THE S_3 MODEL

In this section, we present the calculation of lepton asymmetry in our model and show that for the parameter range of interest from neutrino mixing physics, one can explain the baryon asymmetry of the universe whose present value is given by the WMAP observations[12] to be

$$\frac{n_B}{n_\gamma} = 6.1{\pm}0.2 \times 10^{-10}. \tag{8}$$

Let us start by reminding ourselves of some well known facts about leptogenesis. In the type I seesaw scenario, lepton asymmetry is generated by the out-of-equilibrium decay of the right-handed neutrinos which participate in the seesaw mechanism to give neutrino masses and mixings. Most of the discussion of leptogenesis uses type I seesaw and there have been many papers[4] which have studied its connection to neutrino masses and mixings. In models with both type I[1] and type II seesaw[13] (induced by Higgs triplets[14]), the presence of the triplet Higgs may also contribute to the lepton asymmetry in two ways: either the decay of one or more triplets[15] or the decay of right-handed neutrino with triplets running in the loop[16][17]. Our model involves both type I and type II seesaw; however, it turns out that the first contribution (i.e. the one from triplet decay) is highly suppressed and only the lightest right-handed neutrino(sneutrino) decay is important, which we compute below.

The asymmetry from the decay of the right-handed neutrino ν_{Ri} into a lepton(slepton) and a Higgs(Higgsino) is given by:

$$\varepsilon_i = \frac{\Gamma[\nu_{Ri} \to lH(\tilde{l}\tilde{H})] - \Gamma[\nu_{Ri} \to \bar{l}H^*(\tilde{\bar{l}}\tilde{H}^*)]}{\Gamma[\nu_{Ri} \to lH(\tilde{l}\tilde{H})] + \Gamma[\nu_{Ri} \to \bar{l}H^*(\tilde{\bar{l}}\tilde{H}^*)]}, \tag{9}$$

and we also have the sneutrino $\tilde{\nu}_{Ri}$ decay asymmetry, which we denote as $\tilde{\varepsilon}_i$. If one ignores the supersymmetry breaking effects, one has $\varepsilon_i = \tilde{\varepsilon}_i$.

In the basis where right-handed neutrinos mass matrix is diagonal, the decay asymmetry of right-handed neutrino from type I contribution is given by[18]

$$\varepsilon_i^I = -\frac{1}{8\pi}\frac{1}{[Y'_\nu Y'^\dagger_\nu]_{ii}} \sum_j \mathrm{Im}[\mathrm{Y}'_\nu \mathrm{Y}'^\dagger_\nu]^2_{\mathrm{ij}} F(\frac{M_j^2}{M_i^2}), \tag{10}$$

where $F(x) = \sqrt{x}(\frac{2}{x-1} + \ln[\frac{1+x}{x}])$ and for $x \gg 1$, $F(x) \simeq \frac{3}{\sqrt{x}}$.

The type II contribution has been calculated and is given in Ref. [16][17] to be

$$\varepsilon_i^{II} = \frac{3}{8\pi}\frac{\mathrm{Im}[Y'_\nu f^* Y'^T_\nu \mu]_{ii}}{[Y'_\nu Y'^\dagger_\nu]_{ii} M_i} \ln(1 + \frac{M_i^2}{M_T^2}), \tag{11}$$

where $\mu \equiv \lambda M_T$ and λ is the coupling between triplet and two doublets in the superpotential. In general λ is complex, but its phase can be absorbed by rescaling phases of every elements of matrix f with same amount. We will treat it real in our discussion.

The total contribution to the lepton asymmetry then becomes

$$\varepsilon_i = \varepsilon_i^I + \varepsilon_i^{II}. \tag{12}$$

In our model, the lightest right-handed neutrino is ν_{R2}, and we will take $i = 2$.

The generated $B - L$ asymmetry can be written as

$$Y_{B-L} \equiv \frac{n_{B-L}}{s} = -\eta(\varepsilon_2 Y^{EQ}_{\nu_{R2}} + \tilde{\varepsilon}_2 Y^{EQ}_{\tilde{\nu}_{R2}}) \tag{13}$$

where

$$\begin{aligned} Y^{EQ}_{\nu_{R2}} &= \frac{n^{EQ}_{\nu_{R2}}}{s} = \frac{3}{4}\frac{45\zeta(3)}{\pi^4 g_{*s}} \\ Y^{EQ}_{\tilde{\nu}_{R2}} &= \frac{n^{EQ}_{\tilde{\nu}_{R2}}}{s} = \frac{45\zeta(3)}{\pi^4 g_{*s}}, \end{aligned} \tag{14}$$

g_{*s} is the effective degree of freedom contributing to entropy s with value 228.75 in MSSM, and η is the efficiency factor for leptogenesis. Ignoring the SUSY breaking effect, we have $\varepsilon_2 = \tilde{\varepsilon}_2$ and Y_{B-L} can be simplified as

$$Y_{B-L} = -\frac{7}{4}\frac{45\zeta(3)}{\pi^4 g_{*s}}\eta\varepsilon_2. \tag{15}$$

Lepton number asymmetry produced by decay of right-handed neutrino(sneutrino) can be converted to baryon number asymmetry by sphaleron effect. The baryon number is related to the $B - L$ asymmetry Y_{B-L} via

$$Y_B = wY_{B-L}, \tag{16}$$

where $w = \frac{8N_F+4N_H}{22N_F+13N_H}$ with N_F as generations of fermions and N_H as the number of the Higgs doublet. In MSSM, $N_F = 3$ and $N_H = 2$, one has $w = \frac{8}{23}$. Putting all this together, we get the baryon to photon ratio to be

$$\frac{n_B}{n_\gamma} \simeq 7.04 Y_B = -1.04 \times 10^{-2}\varepsilon_2\eta. \tag{17}$$

The efficiency factor η can be calculated by solving a set of coupled Boltzmann equations(See for example Refs.[19][21]). We assume that to a good approximation the efficiency factor depends only on a mass parameter usually called the effective mass and the initial abundance of the right-handed neutrino(sneutrino). We also use the result for η in type I seesaw scenario. In our model, the effective mass for both the cases discussed below, is given by

$$\tilde{m}_2 = \frac{[Y_\nu Y_\nu^\dagger]_{22} v^2 \sin^2\beta}{M_2} = \frac{2h^2 v^2 \sin^2\beta}{M_2} \simeq \sqrt{\Delta m_A^2} \simeq 0.05\text{eV}, \tag{18}$$

which is larger than the equilibrium neutrino mass $m_* = \frac{16\pi^{5/2}\sqrt{g}_*}{3\sqrt{5}} \frac{v^2 \sin^2\beta}{M_{pl}} \simeq 1.50 \times 10^{-3}$eV, so it is in the strong washout region. In this region, the dependence of efficiency factor on the initial abundance of right-handed neutrino(senutrino) is small[20][21]. We take the approximation formula from Ref.[21] to estimate the efficiency factor for our model

$$\frac{1}{\eta} \simeq \frac{3.3 \times 10^{-3}\text{eV}}{\tilde{m}_2} + (\frac{\tilde{m}_2}{0.55 \times 10^{-3}\text{eV}})^{1.16}, \tag{19}$$

and find $\eta \simeq 5.3 \times 10^{-3}$, which we will use in the calculation of baryon to photon ratio for our model.

IV. EXACT TRI-BIMAXIMAL LIMIT

In this section, we establish the connection between ϵ_ℓ and the low energy phase in the neutrino mixing. In the limit of $|M_{R1,R3}| \to \infty$, light neutrino mass matrix has the form that leads to tri-bimaximal mixing pattern. In this limit, the contributions to lepton asymmetry from the exchange of ν_{R1} and ν_{R3} in the loops are negligible. As far as neutrino masses go, ν_{R2} contribution dominates Δm_A^2 and triplet Higgs has the full contribution to $\Delta m_\odot^2$. The observed values require that $M_T \sim (10^1 - 10^2) M_2$. This triplet can go into loop of the decay of ν_{R2} and its interference with tree level diagram of ν_{R2} decay can generate lepton asymmetry. In this case, Eq.(11) is simplified as

$$\varepsilon_2^{II} = \frac{3}{8\pi} \frac{\text{Im}[Y_\nu f^* Y_\nu^T]_{22}\mu}{[Y_\nu Y_\nu^\dagger]_{22} M_2} \ln(1 + \frac{M_2^2}{M_T^2}). \tag{20}$$

From Yukawa coupling matrices, one easily gets

$$\text{Im}[Y_\nu f^* Y_\nu^T]_{22} = 2h^2(|f_b| \sin\phi_b - |f_a| \sin\phi_a) \tag{21}$$

$$[Y_\nu Y_\nu^\dagger]_{22} = 2h^2. \tag{22}$$

We also have

$$|f_a| = a\frac{M_T}{v^2 \sin^2 \beta\lambda}, |f_b| = b\frac{M_T}{v^2 \sin^2 \beta\lambda} \tag{23}$$

where $a \equiv |a'|$ and $b \equiv |b'|$, and ε_2^{II} can be written as

$$\varepsilon_2^{II} = \frac{3}{8\pi}\frac{(b\sin\phi_b - a\sin\phi_a)M_2}{v^2\sin^2\beta}\frac{M_T^2}{M_2^2}\ln(1+\frac{M_2^2}{M_T^2}). \tag{24}$$

Note that in the tri-bimaximal limit,

$$M_\nu = \begin{pmatrix} ae^{i\phi_a} & be^{i\phi_b} & be^{i\phi_b} \\ be^{i\phi_b} & ae^{i\phi_a} - c & be^{i\phi_b} + c \\ be^{i\phi_b} & be^{i\phi_b} + c & ae^{i\phi_a} - c \end{pmatrix}, \tag{25}$$

which can be diagnolized by U_{TB}

$$U_{TB}^T M_\nu U_{TB} = \begin{pmatrix} ae^{i\phi_a} - be^{i\phi_b} & 0 & 0 \\ 0 & ae^{i\phi_a} + 2be^{i\phi_b} & 0 \\ 0 & 0 & -2c + ae^{i\phi_a} - be^{i\phi_b} \end{pmatrix}. \tag{26}$$

Therefore one of the Majorana phases is given by

$$\varphi_1 \simeq \mathrm{Arc}\sin[\frac{a\sin\phi_a - b\sin\phi_b}{m_1}] \tag{27}$$

up to $O(\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}})$. And for $M_T \geq (10^1 - 10^2)M_2$, one has $\frac{M_T^2}{M_2^2}\ln(1+\frac{M_2^2}{M_T^2}) \simeq 1$. So the lepton asymmetry can be written as

$$\varepsilon_2^{II} \simeq -\frac{3}{8\pi}\frac{m_1 M_2 \sin\varphi_1}{v^2\sin^2\beta}. \tag{28}$$

Thus we see that the Majorana phase φ_1 directly gives the lepton asymmetry, as noted in the introduction. This is the first main result of this paper.

To estimate the value of the baryon to photon ratio, we note that in this case $\varepsilon_2^I \simeq 0$ and $\varepsilon_2 = \varepsilon_2^{II}$, using Eq.(17) and Eq.(28), giving

$$\frac{n_B}{n_\gamma} \simeq 6.1 \times 10^{-10}(\frac{m_1}{2.8\times 10^{-3}\mathrm{eV}})(\frac{M_2}{10^{12}\mathrm{GeV}})(\frac{\sin\varphi_1}{1})(\frac{\eta}{5\times 10^{-3}}), \tag{29}$$

where we take $v = 170$Gev and $\tan\beta = 10$. To get the right range for baryon to photon ratio, the lightest right-handed neutrino mass should be larger than about 10^{12}GeV. Strict lower bound is on the product $m_1 M_2 \geq 2.8$ GeV^2. The thermal production of ν_{R2} requires a reheat temperature of the Universe after inflation be $T_{reh} \gtrsim 10^{12} - 10^{13}$GeV.

If we take as upper bound on M_2 to be 10^{14}GeV required to fit the atmospheric neutrino data, to get right baryon to photon ratio, we have to have a lower bound of $m_1 \sim 10^{-5}$eV. On the other hand, if we take $M_2 \sim 10^{14}$GeV and $m_1 \sim 10^{-3}$eV, we get the lower bound of $\sin\varphi_1$ as $\sim 10^{-2}$.

V. DEPARTURE FROM TRI-BIMAXIMAL MIXING AND NEW CONTRIBUTION TO LEPTOGENESIS

In this section, we consider the case when we relax the mass constraint on the right-handed neutrinos and assume that $|M_{R2}| < |M_{R3}| \ll |M_{R1}|$. This will lead to departures from the exact tri-bimaximal mixing pattern[22]. In this case, there are three independent phases as noted above.

While the type II contribution to neutrino mass matrix in this case remains the same as in the exact tri-bimaximal case, the type I contribution changes and is given by

$$M_I = -M_D^T M_{\nu R}^{-1} M_D = -\begin{pmatrix} \sigma e^{i\phi_3} & 0 & -\sigma e^{i\phi_3} \\ 0 & c & -c \\ -\sigma e^{i\phi_3} & -c & c+\sigma e^{i\phi_3} \end{pmatrix}, \tag{30}$$

where $c \equiv \frac{h^2}{M_2} v^2 \sin^2\beta$ and $\sigma \equiv \frac{h^2}{M_3} v^2 \sin^2\beta$.

Combining the contributions from type I and type II, the light neutrino mass matrix is found to be

$$M_\nu = \begin{pmatrix} ae^{i\phi_a} - \sigma e^{i\phi_3} & be^{i\phi_b} & be^{i\phi_b} + \sigma e^{i\phi_3} \\ be^{i\phi_b} & ae^{i\phi_a} - c & be^{i\phi_b} + c \\ be^{i\phi_b} + \sigma e^{i\phi_3} & be^{i\phi_b} + c & ae^{i\phi_a} - c - \sigma e^{i\phi_3} \end{pmatrix}. \tag{31}$$

To diagnolize M_ν, we first consider $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$. The off-diagonal elements of $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$ are all zeros except $1-3$ and $3-1$ entries,

$$[U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}]_{13} = \sqrt{3}\sigma(ce^{-i\phi_3} + \sigma - a\cos(\phi_3 - \phi_a) + b\cos(\phi_3 - \phi_b)). \tag{32}$$

To further diagnolize $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$, one needs another rotation in the $1-3$ plane. Because of the normal hierarchical mass spectrum of the light neutrinos, one has $c \gg a \simeq b$, and also $c \gg \sigma$ due to small upper bound of $\sin\theta_{13}$ value. In these approximation, the unitarity matrix in $1-3$ plane is

$$V = \begin{pmatrix} 1 & 0 & \xi \\ 0 & 1 & 0 \\ -\xi e^{i\phi_3} & 0 & e^{i\phi_3} \end{pmatrix} \tag{33}$$

where $\xi \simeq \frac{\sqrt{3}\sigma}{4c}$. Now the mixing matrix is given by $U = U_{TB} V$,

$$U=\begin{pmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\xi \\ -\frac{1}{\sqrt{6}}-\frac{e^{i\phi_3}\xi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{e^{i\phi_3}}{\sqrt{2}}-\frac{\xi}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}}+\frac{e^{i\phi_3}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{e^{i\phi_3}}{\sqrt{2}}-\frac{\xi}{\sqrt{6}}\end{pmatrix}. \tag{34}$$

From this mixing matrix, we can read $\tan\theta_{12}=\frac{|U_{12}|}{|U_{11}|}=\frac{1}{\sqrt{2}}$, $\sin\theta_{13}=\sqrt{\frac{2}{3}}\xi$ and $\tan\theta_{23}=\frac{|U_{23}|}{|U_{33}|}\simeq 1-\frac{2\xi}{\sqrt{3}}\cos\phi_3$. Note the correlation between θ_{13} and the departure of θ_{23} from its maximal value. For the Dirac phase, we use the Jarskog invariant[23] to extract it from above mixing matrix $J_{CP}=\mathrm{Im}[U_{11}U_{22}U_{12}^*U_{21}^*]=\frac{1}{8}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}\sin\delta$. From Eq.(34), one can easily get

$$\mathrm{Im}[U_{11}U_{22}U_{12}^*U_{21}^*] \;=\; \frac{\xi}{3\sqrt{3}}\sin\phi_3 \tag{35}$$

$$\frac{1}{8}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}\sin\delta \;=\; \frac{\xi}{3\sqrt{3}}\sin\delta. \tag{36}$$

Therefore we have $\delta\simeq\phi_3$. Remarkably, although this model has three independent CP phase at the seesaw scale, the low energy scale Dirac phase is equal to one of the phases at the high energy scale up to $O(\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}})$. This is independent of the way to assign these three phases.

Coming to the calculation of lepton asymmetry in this case, with $|M_{R2}|<|M_{R3}|\ll|M_{R1}|$ limit, besides the contribution from type II to the lepton asymmetry, we should also consider the contribution from type I. From Eq.(10), we have

$$\varepsilon_2^I=-\frac{1}{8\pi}\frac{1}{[Y_\nu' Y_\nu'^\dagger]_{22}}\mathrm{Im}[Y_\nu' Y_\nu'^\dagger]_{23}^2 F(\frac{M_3^2}{M_2^2}), \tag{37}$$

and $Y_\nu'=U_R^\dagger Y_\nu$, where U_R is to diagnolize the right-handed neutrino mass matrix.

In the two light right-handed neutrinos limit, the phase of the mass of the heaviest right-handed neutrino is irrelevant to the lepton asymmetry and one can take $U_R=\mathrm{diag}(1,1,e^{i\phi_3/2})$. Therefore we have $[Y_\nu' Y_\nu'^\dagger]_{23}=-h^2e^{i\phi_3/2}, [Y_\nu' Y_\nu'^\dagger]_{22}=2h^2$ and $F(\frac{M_3^2}{M_2^2})\simeq 3\frac{M_2}{M_3}$, and plugging them into Eq.(37), we get

$$\varepsilon_2^I=-\frac{3}{8\pi}\frac{h^2}{2}\sin\phi_3\frac{M_2}{M_3} \tag{38}$$

Notice that $\delta\simeq\phi_3$, $\sin\theta_{13}=\sqrt{\frac{2}{3}}\xi=\frac{\sqrt{2}}{4}\frac{M_2}{M_3}$, $\Delta m_A^2\simeq 4c^2$ and $c=\frac{h^2}{M_2}v^2\sin^2\beta$, one can rewrite ε_2^I as function of the low energy scale observables,

$$\varepsilon_2^I\simeq-\frac{3}{8\pi}\frac{\sqrt{\Delta m_A^2}M_2}{\sqrt{2}v^2\sin^2\beta}\sin\delta\sin\theta_{13}. \tag{39}$$

Combining the contribution from ε_2^{II} given in Eq.(28), we have

$$\varepsilon_2 = \varepsilon_2^{II} + \varepsilon_2^{I} \simeq -\frac{3}{8\pi}\frac{M_2}{v^2\sin^2\beta}[\sqrt{\frac{\Delta m_A^2}{2}}\sin\delta\sin\theta_{13} + m_1\sin\varphi_1] \tag{40}$$

We again see that the phases in the leptogenesis formula are the same phases in the neutrino mixing matrix- one Dirac and one Majorana. This is the second main result of our paper. In this case also one can get the right value for the baryon to photon ratio by choosing the M_2 masses.

VI. CONCLUSION

In conclusion, we have shown that in a model for tri-bimaximal neutrino mixing derived from an S_3 permutation symmetry among lepton generations, the observable neutrino phases at low energies are directly responsible for the origin of matter (up to small corrections of order $\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}}$)Therefore, a measurement of the low energy neutrino phase in this model will provide a direct understanding of the high temperature early universe phenomenon of the origin of matter. This model is especially interesting in view of the fact that tri-bimaximal mixing pattern very closely resembles current experimental observations. Measurement of θ_{13} and θ_{23} can provide test of the tri-bimaximal mixing. If this pattern gets confirmed, experimental search for leptonic phases will become a matter of deep interest since it may hold the key to a fundamental mystery of cosmology.

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[1] P. Minkowski, Phys. Lett. B **67**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity* (P. van Nieuwenhuizen et al. eds.), North Holland, Amsterdam, 1980, p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; S. L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons* (M. Lévy et al. eds.), Plenum Press, New York, 1980, pp. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[2] M. Fukugita and T. Yanagida, Phys. Lett. **74 B**, 45 (1986).

[3] V. Kuzmin, V. Rubakov and M. Shaposnikov. Phys. Lett. **185B**, 36 (1985).

[4] For an incomplete list of papers relating neutrino mixings to leptogenesis, see M.S. Berger and B. Brahmachari, Phys. Rev. **D 60**, 073009 (2000); M.S. Berger, Phys. Rev. **D 62**, 013007 (2000); D. Falcone, F. Tramontano, Phys. Rev. **D 63**, 073007 (2001); Phys. Lett. **B 506**, 1 (2001); D. Falcone, Phys. Rev. **D 65**, 077301 (2002); Phys. Rev. **D 66**, 053001 (2002); J. Ellis, M. Raidal, and T. Yanagida, Phys. Lett. **B 546**, 228 (2002); W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B **643**, 367 (2002); S. Davidson, A. Ibarra, Nucl. Phys. **B 648**, 345 (2003); M. Hirsch, S.F. King, Phys. Rev. **D 64**, 113005 (2001); G.C. Branco *et al.*, Nucl. Phys. **B 640**, 202 (2002); M.N. Rebelo, Phys. Rev. **D 67**, 013008 (2003); B. Dutta and R. N. Mohapatra, Phys. Rev. D **68**, 113008 (2003); S. Davidson and A. Ibarra, J. Phys. G **29**, 1881 (2003); E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP **0309**, 021 (2003); S. F. King, Phys. Rev. D **67** (2003) 113010; G.C. Branco, R. Gonzalez Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, Phys. Rev. D **67**, 073025 (2003); S. Pascoli, S. T. Petcov and W. Rodejohann, Phys. Rev. D **68**, 093007 (2003); V. Barger, D. A. Dicus, H. J. He and T. J. Li, Phys. Lett. B **583**, 173 (2004); S. Chang, S. K. Kang and K. Siyeon, Phys. Lett. B **597**, 78 (2004); W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. **315**, 305 (2005); New J.Phys.**6**, 105 (2004);G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685**, 89 (2004); M. C. Chen and K. T. Mahanthappa, Phys. Rev. D **71**, 035001 (2005); R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **615**, 231 (2005); S. Blanchet and P. Di Bari, JCAP **0606**, 023 (2006); arXiv:hep-ph/0607330; X. D. Ji, Y. C. Li, R. N. Mohapatra, S. Nasri and Y. Zhang, arXiv:hep-ph/0605088; A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, arXiv:hep-ph/0605281; JCAP **0604**, 004 (2006); Z. Z. Xing and S. Zhou, arXiv:hep-ph/0607302. For earlier related works, see Z.G.Berezhiani and M.Yu.Khlopov Sov.J.Nucl.Phys. **51**, 739 (1990); Sov.J.Nucl.Phys. **51**, 935 (1990); Sov.J.Nucl.Phys. **52**, 60 (1990).

[5] A. S. Joshipura, E. A. Paschos and W. Rodejohann, JHEP **0108**, 029 (2001); P. H. Frampton, S. L. Glashow and T. Yanagida, Phys. Lett. B **548**, 119 (2002); T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi and M. Tanimoto, Phys. Rev. Lett. **89**, 231601 (2002); K. Bhattacharya, N. Sahu, U. Sarkar and S. K. Singh, arXiv:hep-ph/0607272; Steve King and Antonio Riotto, hep-ph/0609038; S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0609125; G. C. Branco, R. G. Felipe and F. R. Joaquim, arXiv:hep-ph/0609297.

[6] R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **639**, 318 (2006)

[7] L. Wolfenstein, Phys. Rev. **D 18** , 958 (1978); P. F. Harrison, D. Perkins and W. G. Scott, Phys. Lett. **B 530**, 167 (2002); P. F. Harrison and W. G. Scott, Phys. lett. **B 535**, 163 (2002); Z. Z. Xing, Phys. Lett. **B 533**, 85 (2002).

[8] K. S. Babu and X. G. He, hep-ph/0507217; G. Altarelli and F. Feruglio, hep-ph/0512103; B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M. K. Parida,hep-ph/0603059.

[9] C. I. Low and R. R. Volkas, Phys. Rev. D **68**, 033007 (2003);I. de Medeiros Varzielas, S. F. King and G. G. Ross, hep-ph/0512313; W. Grimus and L. Lavoura, JHEP **0601**, 018 (2006); N. Haba, A. Watanabe and K. Yoshioka, hep-ph/0603116; P. Kovtun and A. Zee, hep-ph/0604169.

[10] P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); F. Caravaglios and S. Morisi, hep-ph/0503234; R. Jora, S. Nasri and J. Schechter, hep-ph/0605069; O. Felix, A. Mondragon, M. Mondragon and E. Peinado, hep-ph/0610061.

[11] C. Hagedorn, M. Lindner and R. N. Mohapatra, in preparation.

[12] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.

[13] G. Lazarides, Q. Shafi and C. Wetterich, Nucl.Phys.**B181**, 287 (1981); R. N. Mohapatra and G. Senjanović, Phys. Rev. **D 23**, 165 (1981).

[14] R. E. Marshak and R. N. Mohapatra, VPI-HEP-80/02 *Invited talk given at Orbis Scientiae, Coral Gables, Fla., Jan 14-17, 1980*, Published in Orbis Scientiae 1980; p. 277; J. Schecter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980); T. P. Cheng and L. F. Li, Phys. Rev. D **22**, 2860 (1980).

[15] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998); T. Hambye, E. Ma and U. Sarkar, Nucl. Phys. B **602**, 23 (2001); G. D'Ambrosio, T. Hambye, A. Hektor, M. Raidal and A. Rossi, Phys. Lett. B **604**, 199 (2004); E.J. Chun, S. Scopel, Phys.Lett. **B636**, 278 (2006); hep-ph/0609259; E.J Chun, S.K. Kang, Phys.Rev. **D63**, 097902 (2001).

[16] T. Hambye and G. Senjanovic, Phys. Lett. B **582**, 73 (2004);

[17] S. Antusch and S. F. King, Phys. Lett. B **597**, 199 (2004).

[18] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384**, 169 (1996).

[19] W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000).

[20] W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. **315**, 305 (2005);

[21] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685**, 89 (2004)

[22] F. Plentinger and W. Rodejohann, Phys. Lett. B **625**, 264 (2005); J. D. Bjorken, P. F. Harrison

and W. G. Scott, arXiv:hep-ph/0511201.

[23] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).